**MATHEMATICS SS 3**

**FIRST TERM**

**SCHEME OF WORK**

### FIRST TERM

1. Revision: Indices and Logarithm
2. Surds
3. Surds in relation to Trigonometry
4. Matrices and Determinants
5. Linear and Quadratic Equations
6. Surface Area and Volume of Sphere and Hemispherical shapes
7. Mid-term Test
8. Longitude and Latitude
9. Longitude and Latitude
10. Arithmetic of Finance
11. Revision
12. Examination
13. Vacation

### WEEK ONE

**TOPIC: BASIC CONCEPT & APPLICATION OF LAWS OF INDICES CONTENT:**

Basic Concept of Laws of Indices

Application of Laws of Indices

### Basic Concept of Laws of Indices

A number of the form am where a is a real number, a is multiplied by itself m times

The number a is called the base and the super script m is called the index (plural indices) or exponent.

1. A m x A n = Am + n ——————–Multiplication law

Example: X3 xX2  =( X x Xx X) x (X x X) = X 5

Or X3 x X2 = X3 + 2 = X5

1. Am ÷ An=Am-n  ———————Division law

Example: X6 ÷ X4 = X6-4 = X2

1. (a m )n = amn —————-Power law

Example: (x3)2 = X3 x X3 = X3+3 =X6

Or X3X2 = X6

1. am ÷ am = am-m = a0 =1 am ÷am = am/am = ao = 1

a0………………………………….Zero Index

:. Any number raised to power zero is 1

Example: 3o = 1, co = 1, yo = 1

1. (ab)m =ambm  ————-Product power law

e.g. (2xy)2= 4x2y2

1. Negative index a –m = 1/am ————- Negative Index Example: 2 -1 = ½, and 3 -2 = 1/3 2 = 1/9
2. a1/n  =n√a ————–Root power law

Example **:** 9 ½ =√9=3

27 1/3 =3√27 = 3 ie (3)3 = 3

8. a m/n = (a 1/n) m = (n√a)m ———–Fraction Index or a m/n = (am) 1/n = (n√a)m

Example : 272/3 = 3√27=32=9.

**EVALUATION**

1. 275/3 2. 10000000000

### Application of Laws of Indices

Solve the following

(i) 32 3/5 (ii) 343 2/3 (iii) 64 2/3(iv) 0.001 (v) 14 0

Solution:

1. 32 3/5 = (32 1/5) 3 = (5√32) 3

= 2 3 = 8

1. 343 2/3 = (343 1/3 )2 = (3√343)2

= (7 3)1/3)2

= 72 = 49

1. 64 2/3 = (64 1/3)2 = (4 3)1/3)2 = 4 2 iv) (0.001)3 = (1/100)3 = (1/10)3)3 = (10 -3)3

= 10 -9 = 1/10 9

v) 14 0 = 1

### EVALUATION

1) Simplify the following

a) 216 4/3 b) 25 1.5 c) (0.00001)2

d) d 32 2/5  e) 81 ¾

### WEEKEND ASSIGNMENT

1. Evaluate 3 x  = 1/8 1 (a) 4 (b) -4 (c) -2 (d) 2
2. Simplify 2r5 X 9r3 (a) P2 (b) 2p4 (c) P3 d)18r8
3. Solve 3-y = 243 (a) -5 (b) 5 (c) 3 (d) -3
4. Solve 25-5n = 625 (a) 1/5 (b) 2/5 (c) 1 1/5 (d) – 2/5
5. Simplify (0.0001)2= (a) 10-5 (b) 10 -3 (c) 10-8 (d) 10-10

Theory

1. 163/2 x 82/32. 3X2 x 4X3

321/56X7

### WEEK 2

**TOPIC: INDICIAL/EXPONENTIAL EQUATION CONTENT:**

Exponential Equation of Linear Form

Exponential Equation of Quadratic Form

Under exponential equation, if the base numbers of any equation are equal, then the power will be equal&vice versa.

### Exponential Equation of Linear Form

Solve the following exponential equations

a) (1/2) x = 8 b) (0.25) x+1 = 16

c) 3x = 1/81 d) 10 x = 1/0.001

e) 4/2x = 64 x

Solution

a) (1/2) X = 8

(2 -1) x = 2 3

2 –x = 2 3

-x = 3

X = – 3

b) (0.25) x+1 = 16

(25/100) x+1 = 16

(1/4) x+1 = 4 2

(4 -1) x+1 = 4 2

* x – 1 = 2
* x = 2 + 1
* x = 3

X = – 3

1. 3x = 1/81

3x = 1/34

3x = 3 -4

X = -4

1. 10x = 1/0.001 10 x = 1000 10 x = 10 3

10x = 10 3

X = 3

e) 4/2x = 64 x

4÷2x = 64 x

22 ÷2x = 64 x

2 2-x = (2 6) x

2 2-x = 2 6x

## 2-x = 6x

2=6x+x

2 = 7x

Divide both sides by 7

2/7 = 7x/7

## X = 2/7

### EVALUATION

Solve the following exponential equations

a) 2 x = 0.125 b) 25 (5x) = 625 c) 10 x = 1/100000

### Exponential Equation of Quadratic Form

Some exponential equation can be reduced to quadratic form as can be seen below.

Example : Solve the following equations.

1. 2 2x – 6 (2 x) + 8 = 0
2. 5 2x + 4 X 5 x+1 – 125 = 0
3. 3 2x – 9 = 0

Solution

a) 2 2x – 6 (2 x) + 8 = 0

(2 x)2 – 6 (2 x) + 8 = 0

Let 2 x = y.

Then y2 – 6y + 8 = 0

Then factorize

Y 2 – 4 y – 2y + 8 = 0

Y (y – 4) -2 (y -4) = 0

(y -2) (y – 4) = 0

Y – 2 = 0 or y – 4 = 0

Y = 2 or y= 4

Y = 2, 4

# Since 2 x = y, and y = 2

[2 x = 2 2 x = 2 1](#_Toc56978)

[x = 1](#_Toc56979)

[Since 2 x = y and y = 4](#_Toc56980)

[2 x = 4](#_Toc56981)

2 x = 2 2  N = 2

X = 1 and 2

b) 5 2x + 4 X 5 x+1 – 125 = 0

(5 x) 2 + 4 X (5 x X 5 1) – 125 = 0

Let 5 x = p

P 2 + 4 X (p X 5) – 125 = 0

P2 + 4 (5p) – 125 = 0

P2 + 20p – 125 = 0

Then Factorise p2 + 25p – 5p – 125 = 0

P (p + 25) -5 (p + 25) = 0

(p – 5) (p + 25) = 0

P – 5 = 0 p + 25 = 0

P = 5 or p = – 25

Since 5x = p, p = 5

5 x  = 5 1

X = 1

5x = -25 (Not simplified)

1) 3 2x – 9 = 0

(3 x) 2 – 9 = 0 Let 3 x  = a a 2 – 9 = 0 a 2 = 9

a = ±√9 a = ± 3 a = 3 or – 3

Since 3 x  = a, when a = 3

3 x  = 3 1

X = 1

Since 3x = a, when a = -3

3 x  = – 3 (Not a solution)

### EVALUATION

Solve the following exponential equations.

1. 2 2x+ 1 – 5 (2 x) + 2 = 0
2. 3 2x – 4 (3 x+1) + 27 = 0

### WEEKEND ASSIGNMENT

1. Solve for X : (0.25) X + 1 = 16 (a) -3 (b) 3 (d) 4 (d) -4
2. Solve for X : 3(3)X = 27 (a) 3 (b) 4 (c) 2 (d) 5
3. Solve the exponential equation : 22X + 2X+1 – 8 = 0 (a) 1 (b) 2 (c) 3 (d) 4
4. The second value of X in question 3 is (a) -1 (b) 1 (c) 2 (d) It has no solution 5. Solve for X : 10-X = 0.000001 (a) 4 (b) 6 (c) -6 (d) 5

### Theory

Solve the following exponential equations

(1.) (3X)2 + 2(3X )– 3 = 0 (2). 52X+1– 26(5X) + 5 = 0

### WEEK THREE

**TOPIC: LOGARITHM- SOLVING PROBLEMS BASED ON LAWS OF LOGARITHM CONTENT:**

Logarithm of numbers (Index & Logarithmic Form)

Laws of Logarithm

Logarithmic Equation Change of Base

### Logarithm of numbers (Index & Logarithmic Form)

The logarithm to base **a** of a number **P**, is the index **x** to which **a** must be raised to be equal to **P**.

Thus if P = ax, then x is the logarithm to the base **a**of **P**. We write this as x = log a P. The relationship logaP = x and ax=P are equivalent to each other.

ax=P is called the index form and logaP = x is called the logarithm form

Conversion From Index to Logarithmic Form

Write each of the following index form in their logarithmic form

a) 26 = 64 b) 251/2 = 5 c) 44= 1/256

Solution

1. 26 = 64 Log2 64 = 6
2. 251/2 = 5 Log255=1/2
3. 4-4= 1/256

Log41/256 = -4

Conversion From Logarithmic to Index form.

a) Log2128 = 7 b) log10 (0.01) = -2

c) Log1.5 2.25 = 2

Solution

1. Log2128 = 7

27 = 128

1. Log10(0.01) = -2

10-2= 0.01

1. Log1.5 2.25 = 21.52 = 2.25

**Laws of Logarithm**

1. let P = bx, then logbP = x

Q = by, then logbQ = y

PQ = bx X by = bx+y (laws of indices)

LogbPQ = x + y :. LogbPQ = logbP + LogbQ

1. P÷Q = bx÷by = bx+y  LogbP/Q = x –y :. LogbP/Q = logbP – logbQ
2. Pn= (bx)n = bxn  Logbpn = nbx :. LogPn = logbP
3. b = b1 :. Logbb = 1
4. 1 = b0

Logb1 = 0

EXAMPLE – Solve each of the following:.

1. Log327 + 2log39 – log354
2. Log313.5 – log310.5
3. Log28 + log23
4. given that log102 = 0.3010 log103 = 0.4771 and log105 = 0.699 find the log1064 + log1027 Solution

a) Log327 + 2 log39 – log354

= log3 27 + log3 92 –log354

= log3 (27 X 92/54)

= log3 (271 X 81/54) = log3 (81/2)

= log3 34/log32

= 4log3 3 – log3 2

4 X (1) – log3 2 = 4 – log3 2

= 4 – log3 2

1. log3 13.5 – log3 10.5

= log3 (13.5)- Log310.5 = log3 (135/105)

= log3 (27/21) = log3 27 – log3 21

= log3 3 3 – log3 (3 X 7)

= 3log3 3 – log3 3 -log37

= 2 – Log3 7

1. Log28 + Log33

= log223+ log33

= 2log22 + log33

2+1=3

1. log10 64 + log10 27 log10 26 + log1033  6 log10 2 + 3 log10 3

6 (0.3010) + 3(0.4771)

1.806 + 1.4314 = 3.2373.

### EVALUATION

1. Change the following index form into logarithmic form.

(a) 63= 216 (b) 33 = 1/27 (c) 92 = 81

2. Change the following logarithm form into index form.

(a) Log88 = 1 (b) log ½¼ = 2 3. Simplify the following

1. Log512.5 + log52
2. ½ log48 + log432 – log42
3. Log381

4. Given that log 2 = 0.3010, log3 0.477

Log105 = 0.699, find the log10 6.25 + log10

### Logarithmic Equation

Solve the following equation

1. Log10 (x2 – 4x + 7) = 2
2. Log8 (r2 – 8r + 18) = 1/3

Solution

a) Log10 (x2 – 4x + 7) = 2

X2 – 4x + 7 = 102 (index form)

X2 – 4x + 7 = 100

X2 – 4x + 7 – 100 = 0

X2 – 4x – 93 = 0

Using quadratic formular

= – b ±√b2– 4ac

2a a = 1, b = -4, c = – 93 x = – (- 4) ± √(- 4) 2 – 4 X 1 X (- 93)

## 2 X 1

= + 4 ± √16 + 372

2

= + 4 ± √388/2 = x = 4 +√ 388/2 or 4 – √388/2 x = 11.84 or x = – 7.85

2) Log8 (x2 – 8x + 18) =81/3

X2 – 8x + 18 = 81/3  X2 – 8x + 18 = (2)3X1/3

X2 – 8x + 18 =2

X2 – 8x 18 – 2 = 0

X2 – 8x + 16 = 0

X2 – 4x – 4x + 16 = 0

X(x – 4) -4 (x – 4) = 0

(x – 4) (x – 4) = 0

(x – 4) twice

X = + 4 twice

### Change of Base

Let logbP = x and this means P = bx

LogcP = logcbx = x logcb

If x logcb = logcP

X = logcP

Logc b

:. LogcP = logcP

Logcb

Example : Shows that logab X logba = 1

Logab = logcb

Logca

Logba = logca

Logcb

:. Logab X logba = logcb X logca

Logca + logcb= 1

### EVALUATION

Solve the following logarithm equation.

Log3 (x2 + 7x + 21) = 2

Log10 (x2 – 3x + 12) = 1

### ASSIGNMENT

1. If log81/64 = x, find the value of x (a) 2 (b) 1 (c) -3 (d) -4.

1. Solve 9(1 – x) = (1/27) x+1 (a) -5 (b) -1 (c) 1 (d) ½
2. Simplify log7 49 (a) 1/7 (b) 2 (c) 7 (d) log 1/7
3. Solve the equation log216 = x (a) 8 (b) 4 (c) 2 (d) 2
4. Convert 52 = 25 into logarithm form (a) log525 = 2 (b) log 255 = 2 (c) log225 = 5 (d)

None of the above

### Theory

1. Find the value of x for which log10 (4x2 + 1) -2 log10 x – log10 2 = 1 is valid.
2. Solve the logarithmic equation: Log4 (x2 + 6x + 11) = ½

**TOPIC: LOGARITHM OF NUMBERS LESS THAN ONE**

### CONTENTS

Standard forms

Logarithm of numbers greater than one

Multiplication and divisions of numbers greater than one using logarithm

Using logarithm to solve problems with roots and powers (no > 1) Logarithm of numbers less than one.

Multiplication and division of numbers less than one using logarithm Roots and powers of numbers less than one using logarithm

### Standard Forms

Numbers such as 1000 can be converted to its power of ten in the form 10n where n can be term as the number of times the decimal point is shifted to the front of the first significant figure i.e. 10000 = 104

Number Power of 10

1. 102
2. 101
3. 100

1. 10-3

1. 10-1

Note: One tenth; one hundredth, etc are expressed as negative powers of 10 because the decimal point is shifted to the right while that of whole numbers are shifted to the left to be after the first significant figure.

A number in the form A x 10n, where A is a number between 1 and 10 i.e. 1<A<10 and n is an integer is said to be in ***standard form*** e.g. 3.835 x 103 and 8.2 x 10-5 are numbers in standard form.

Examples : Express the following in standard form

1. 7853
2. 382
3. 0.387
4. 0.00104

Solutions

1. 7853 = 7.853 x 103 2. 382 = 3.82 x 102

1. 0.387 = 3.87 x 10-1
2. 0.00104 = 1.04 x 10-3

### Logarithm of numbers greater than one

Base ten logarithm of a number is the power to which 10 is raised to give that number e.g.

628000 = 6.28 x105

628000 = 100.7980 x 105

= 100.7980+ 5

= 105.7980

Log 628000 = 5.7980

IntegerFraction (mantissa)

If a number is in its standard form, its power is its integer i.e. the integer of its logarithm e.g. log 7853 has integer 3 because 7853 = 7.853 x 103

Examples: Use tables (log) to find the complete logarithm of the following numbers.

(a) 80030 (b) 8 (c) 135.80

1. 80030 = 4.9033
2. 8 = 0.9031
3. 13580 = 2.1329

### Multiplication and Division of number greater than one using logarithm

To multiply and divide numbers using logarithms, first express the number as logarithm and then apply the addition and subtraction laws of indices to the logarithms. Add the logarithm when multiplying and subtract when dividing.

Examples: Evaluate using logarithm.

1. 4627 x 29.3
2. 8198 ÷ 3.905
3. 48.63 x 8.53

15.39

Solutions

1. 4627 x 29.3

### No Log

4627 3.6653

X 29.3 + 1.4669

Antilog → 1356005.1322

**.**

\ 4627 x 29.3 = **135600**

To find the Antilog of the log 5.1322 use the antilogarithm table:

Check 13 under 2 diff 2 (add the value of the difference) the number is 0.1356. To place the decimal point at the appropriate place, add one to the integer of the log i.e. 5 + 1 = 6 then shift the decimal point of the antilog figure to the right (positive) in 6 places.

= 135600

2. 819.8 x 3.905

### No Log

819.8 2.9137

3.905 0.5916

antilog → 209.9 2.3221

\ 819.8 ÷ 3.905 = **209.9**

3. 48.63 X 8.8.53

15.39

### No Log

48.63 1.6869

8.53 + 0.9309

2.6178 ÷ 15.39 – 1.1872 antilog → 26.95 1.4306

\48.63 ÷ 8.53 = 26.96

15.39

**Evaluation:**

1. Use table to find the complete logarithm of the following:

(a) 183 (b) 89500 (c) 10.1300 (d) 7

2 Use logarithm to calculate.

3612 x 750.9

113.2 x 9.98

**Using logarithm to solve problems with powers and root (nos. greater than one).**

Examples:

Evaluate

1. 3.533 (b) 4 40000 (c) 94100 x 38.2

5.683 x 8.14 (2s.f)

**Solution**

**No. Log\_\_\_\_\_**

3.533 0.5478 x 3

44.00 1.6434

\ 3.533 = 44.00

1. 4 40000

**No. Log\_\_\_\_\_**

4 4000 3.6021 ÷ 4

#### 7.952 0.9005 \ 4 4000 = 7.952

(c) 94100 x 38.2

5.6833 x 8.14

Find the single logarithm representing the numerator and the single logarithm representing the denominator, subtract the logarithm then find the anti log.

(Numerator – Denominator).

### No Log

94100 4.9736 ÷ 2 = 2.4868

38.2 1.5821

Numerator **4.0689** → 4.0689

5.683 0.7543 x 3 = 2.2629

8.14 0.9106

### Denominator 3.1735 → 3.1735

**7.859 0.8954**

\94100 x 38.2 = 7.859

5.683 x 8.14

### ~ 7.9 (2.sf)

**Logarithm of number less than one.**

To find the logarithm of number less than one, use negative power of 10 e.g.

0.037 = 3.7 x 10-2  10 0.5682 x 10-2

1. 0.5682 + (-2)

10-2 5682

Log 0.037 = 2 . 5682

2 . 5682

Integer decimal fraction (mantissa)

Example: Find the complete log of the following.

(a) 0.004863 (b) 0.853 (c) 0.293

Solution

Log 0.004863 = 3.6369

Log 0.0853 = 2.9309

Log 0.293 = 1.4669

### Evaluation

1. Find the logarithm of the following:

(a) 0.064 (b) 0.002 (c) 0.802

2. Evaluate using logarithm.

95.3 x 318.4

1.295 x 2.03

**Using logarithm to evaluate problems of Multiplication, Division, Powers and roots with numbers less than one.**

Examples:

1. 0.6735 x 0.928
2. 0.005692 ¸ 0.0943
3. 0.61043
4. 4 0.000
5. 3 0.06642

Solution

1. 0.6735 x 0.928

**No. Log.\_\_\_**

0.6735 1.8283

0.928 1.9675

**0.6248 1.7958**

### \ 0.6735 x 0.928 = 0.6248

1. 0.005692 ¸ 0.0943

**No Log**

0.005692 3.7553

¸ 0.0943 2.9745

**0.06037 2.7808**

1. 0.61043

**No Log\_\_\_\_\_**

0.61043 1.7856 x 3

0.2274 1.3568

\ 0.61043 = **0.2274**

\ 0.005692 ¸ 0.943 = 0.6037

1. 4 0.00083

**No. Log.\_\_\_\_\_**

4 0.00083 4.9191 ¸ 4

0.1697 1.2298

\ 4 0.06642 = **0.1697**

5. 3 0.6642

**No. Log.\_\_\_\_\_\_\_\_\_\_\_\_**

3 0.6642 2.8223 ¸ 3

2.1 + 1 + 0.8223 ¸ 3

3 + 1 .8223 ¸ 3

1 + 0.6074

0.405 1.6074

3 0.6642 = **0.405**

Note: 3 cannot divide 2 therefore subtract 1 from the negative integer and add 1 to the positive decimal fraction so as to have 3 which is divisible by 3 without remainder.

**Evaluation:**

1. Evaluate

5 (0.1684)3

1. 6.28 x 304

981

### ASSIGNMENT

Use table to find the log of the following:

1. 900 (a) 3.9542 (b) 1.9542 (c) 2.9542 (d) 0.9542
2. 12.34 (a) 3.0899 (b) 1.089 (c) 2.0913 (d)

1.0913

1. 0.000197 (a) 4.2945 (b) 4.2945 (c) 3.2945 (d)

3.2945

1. 0.8 (a) 1.9031 (b) 1.9031 (c) 0.9031 (d)

2.9031

1. Use antilog table to write down the number whose logarithms is 3.8226. (a) 0.6646 (b) 0.06646 (c) 0.006646 (d) 66.46

### Theory

Evaluate using logarithm.

1. 23.97 x 0.7124

3.877 x 52.18

1. 3 69.52 – 30.52

**WEEK TWO**

**TOPIC: SURDS COTENT:**

Rules of surds

Basic Form of Surds Similar Surds

Conjugate Surds

Simplification of Surds

Additional & Subtraction of Surds

Multiplication and Division of Surds

Rationalization of Surds Equality of Surds

### Rules of Surds

Surds are irrational numbers. They are the root of rational numbers whose value can not be expressed as exact fractions. Examples of surds are: √2, √7, √12, √18, etc.

1. √(a X b ) = √a X √ b
2. √(a / b ) = √a / √b
3. √(a + b ) ≠ √a + √b
4. √(a – b ) ≠ √a – √b

### Basic Forms of Surds

√a is said to be in its basic form if A does not have a factor that is a perfect square.

E.g. √6, √5, √3, √2 etc. √18 is not in its basic form because it can be broken into √ (9×2) = 3√2. Hence 3√2 is now in its basic form.

### Similar Surds

Surds are similar if their irrational part contains the same numerals e.g.

1. 3√n and 5√ n
2. 6√2 and 7√2

### Conjugate Surds

Conjugate surds are two surds whose product result is a rational number.

(i)The conjugate of √3 – √5 is √3 + √5

The conjugate of -2√7 + √3 is -2√7 – √3

In general, the conjugate of √x + √y is √x – √y

The conjugate of √x – √y = √x + √y

### Simplification of Surds

Surds can be simplified either in the basic form or as a single surd.

Examples 1. Simplify the following in its basic form (a) √45 (b) √98

Solution

1. √45 = √ (9 x 5) = √9 x √5 = 3√5
2. √98 = √ (49 x 2) = √49 x √2 = 7√2

Example 2. Simplify the following as a single surd (a) 2√5 (b) 17√2

Solution

1. 2√5 = √4 x √5 = √ (4 x 5) = √20
2. 17√2 = √289 x √2 = √ (289 x 2) = √578

### Addition and Subtraction of Surds

Surds in their basic forms which are similar can be added or subtracted.

Example: Evaluate the following

(a)√32 + 3√8 (b) 7√3 – √75 (c) 3√48 – √75 + 2√12

Solution

(√32 + 3√8

=√ (16 x 2) + 3√ (4 x 2)

=4√2 + 6√2

= 10√2

1. 7√3 – √75

= 7√3 – √ (25 x 3)

=7√3 – 5√3

=2√2

1. 3√48 – √75 + 2√12

= 3√ (16 x 3) – √ (25 x 3) + 2√ (4 x 3)

= 12√3 – 5√3 + 4√3

= 11√3

### EVALUATION

1. Simplify the following **(a)**5 √ 12 – 3 √ 18 + 4 √72 + 2 √75 (b) 3√2 – √32 + √50 + √98
2. Multiply the following by their conjugate (a) √3 -2√5 (b) 3√2 + 2√3

### Multiplication and Division of Surds

Example: Evaluate the following (a) √45 x √28 (b) √24 / √50

Solution

√45 x √28

= √ (9 x 5) x √ (4 x 7)

= 3√5 x 2√7

= 3 x 2 x √ (5 x 7)

= 6√35

(b)√24 / √50

= √ (24 / 50)

= √ (12 / 25)

= √12 / √25

= √ (4 x 3) / 5

= 2√3 / 5

### Surds Rationalisation

Rationalisation of surds means multiplying the numerator and denominator by the denominator or by the conjugate of the denominator.

Example: Evaluate the following (a) 6 / √3 (b) 3

√3 + √2

Solution

6 / √3 (b) 3

= 6 x √3 √3 + √2

√3 x √3 = 3 (√3 – √2)

= 6√3 (√3 + √2) (√3 – √2)

3 = 3√3 – 3√2

= 2√3 (√3)2 – (√2)2

= 3√3 – 3√2

3 – 1

= 3√3 – 3√2

1

= 3( √3 -√2)

### Equality of Surds

Given two surds i.e P + m and q + n if P + m = q + n, then

P – q = n – m the L.H.S

Of the equation is a rational number while the L.H.S and R.H.S can only be equal of they are both equal to zero (0)

P – q = 0

:. P = q and n – m = 0 i.e.

N = m

Examples – Find the square root of the following?

a) 7 + 2 10 b) 14 – 4 6

Solution

(a) Let the square root of 7 + 2 10 be m + n

( m + n) 2 = 7 + 2 10

M +2 m n+ n = 7 + 2 10

M + n = 7 \_\_\_\_\_ (1)

2 m n = 2 10

M n = 10

Squarely both surds we have

M n = 10 \_\_\_\_\_\_\_(ii)

M + n = 7 \_\_\_\_\_\_ (i)

M n = 10 \_\_\_\_\_\_\_ (ii)

From equation (1) m = 7 – n

Put m in (ii) we have

(7 – n) n = 10

7n – n2 = 10

In sum; n2 – 7n + 10 = 0 n2 – 2n – 5n + 10 =0 n (n – 2) – 5 (n – 2) = 0

(n -5) (n – 2) = 0 n = 5 or 2 m = 7 – 2, where n = 2

m = 5,

m = 7 – 5 , when n = 5

m = 2 m= 5 or 2

The square root of 7 + 10 are 5 + 2 twice.

(b) Let the square root of 14 – 4 6 be p – Q

The ( p – Q)2 = 14 – 4 6

P – 2 pQ + Q = 14 – 4 6

P +Q = 14 ……………………………(1)

-2 pQ = ~~4~~ 6

-2 – 2

PQ = 2 6 (squaring both sides)

PQ = = (2 6 )2

PQ = 4 x 6 ……………………………….. (11)

P +Q = 14 ………………………………… (1)

PQ = 24 ……………………………………… (11)

From equation……………… (1) p = 14 – Q

Sub for p in equation ………………… (11)

(14 –Q ) Q = 24

14 Q – Q 2 = 24

In turn we have :

Q2 – 14 Q + 24 = 0

Q2 – 12Q– 2Q + 24 = 0

Q (Q -12) – 2 (Q – 12) = 0 Q = 2 or 12 if p = 14 – Q ,when Q= 12 p = 14 – 12

p = 2 if p = 14 – Q, when q = 2

p = 14- 2

= 12

12 – 2 = 2 3 – 2 and

2 – 12 = 2 – 2 3

### EVALUATION

1.Express 3√2 – √3in the form √mwhere m and n are whole number.

2√3 – √2 √n

2.Express 1 in the form p√5 + q√3, where p and q are rational numbers.

√5 +√3

### ASSIGNMENT

1.Expand (3√2 – 1)(3√2 + 1) (a) 16 (b) 20 (c) 17 (d) 24

2.Simplify √200 in its basic form (a) 10√2 (b) 5√4 (c) 2√10 (d) 2√50

3.Simplify 9/√3 (a) 3√2 (b) 3√3 (c) 1/3 (d) 2√2

4.Express 3√5 as a single surd (a) √40 (b) √55 (c) √45 (d) √35implify

5.Simplify √`128 – 4√8 (a) 0 (b) 1 (c) 2 (d) 3

### Theory

1.Express 3√2 – √3in the form √mwhere m and n are whole number.

2√3 – √2 √n

2.Express 1 in the form p√5 + q√3, where p and q are rational numbers.

√5 +√3

1. . Evaluate the following (a) 323/5 (b) 251.5 (c) (0.000001)2 (d) 3432/3  (e) 190
2. . Solve the following exponential equations (a) 2x = 0.125 (b) 3-x = 243 (c) 25x =

625 (d) 10x = 1/0.001 (e) 4/2x =64x

3 . Solve the following exponential equations (a) 22x -6(2x) + 8 = 0 (b) 22x+1 -5(2x) + 2 = 0

(c) 32x – 4(3x+1) +27 = 0 (d) 32x – 9 = 0 (e) 72x – 2 X 7x + 1 = 0

4 . Change each of the following index form to their logarithmic form (a) 26 = 64

(b) 3-3 =1/27 (c) 251/2 =5 (d) 35 = 243 (e) (0.01)2 = 0.0001

5 . Change the following logarithmic form into index form (a) log2128 = 7 (b) log1/2(1/4) =

2 (c) log749 = 2 (d) log51/125 = -3 (e) log51 = 0

6 . Simplify each of the following (a) log327 + 2log39 –log354 (b) 1/2log48 + log432 – log42

(c) log2√8 + log3√3 (d) logxx9 (e) log512.5 + log52

7 . Solve the following logarithmic equations (a) log10(x2 – 4x + 7) =

2 (b) log8(x2 – 8x + 18) = 1/3 (c) log5(x2 – 9) = 0 (d) log4(x2 + 6x + 11) = ½

1. . Use logarithm table to evaluate the following (a) (3.68)2 x 6.705 (b)√0.897 x 3.536

√0.3581 0.00249

1. . Simplify each of the following (a) 2√12 + 3√48 + √75 (b) 4√8 – 2√98 +

√128 (c) (3√2 – 1) (3√2 + 1 )

1. . Express 1 in the formm√5 + n√3where m and n are rational numbers

3√5 + 5√3

**WEEK 4**

**Matrices and determinants: concept, the basic operations of matrices. Identity matrices and equal matrices**

### MATRICES

Matrix is a rectangular array of numbers or elements in a row or column. e.g ( a b ), a b

Elements arranged along the horizontal are called **ROW**. While elements arranged along the vertical is called **COLUMN**.

E,g

5 7 5 7 row 1

8 12 8 12 row 2

Column 1 2

**NOTATION**: A matrix is denoted by capital letter and the elements by small letters with reference to the position of the element. The position is defined in terms of the number of rows and columns.The first indicating the row, the second the column, thus:

a11 a12 a13b11 b12

A = a21 a22 a23B = b21b22 a31 a32 a33

Hence, a21indicates the element in the second row and first column.

**EVALUATION:** Given the matrix,**C**= 6 -5 1 -3 write out the elements occupying the

following,

Positions.C11, C21, C32, C42, C44 andC34 2 -4 8 3

4 -7 -6 5

-2 9 7 -1

***Order of a matrix***: A matrix can be identified or described by its order. In describing a matrix, the number of rows is stated first before the number of columns. E.g 6 2 8 is a 2 x 3 matrix, i.e. order 2 by 3.  **5 7 3**

**BASIC OPERATION OF MATRICES:**

***Addition and subtraction of matrices***: Two or more matrices can be added or subtracted when they are of the same order e.g 2 x 2, 3 x 3 and so on. The sum or subtraction is then determined by adding or subtracting corresponding elements.

If A = a11 a12 B = b11 b12A + B = a11 + b11 a12 + b12 A – B = a11 –b11 a12 –b12  a21 a22 b21 b22 a21+ b21 a22 + b22a21– b21 a22 – b22

Example: Given the matrices below, find I A + B II A – B III B – A .

A = 7 6 5 B = 12 -6 4

9 4 8 2 10 1

I A + B = 19 0 9 II A – B = -5 12 1 III B – A = 5 -12 -1

11 14 9 7 -6 7 -7 6 -7 Addition of matrices is commutative, i.e A + B = B + A but matrix subtraction is not

commutative,

A – B ≠ B – A

**MULTIPLICATION OF MATRICES**:

1. ***Scalar multiplication***: This is the multiplication of a matrix by a single number and it is done by multiplying each element in the matrix by the scalar.

e.g If C = 3 -8 12 find I 2C II -3C

4 5 7

I 2C = 6 -16 24 II – 3C = -9 24 -36

8 10 14 -12 -15 -21

2. ***Multiplication of two matrices***: Two matrices can be multiplied together only when the number of columns in the first is equal to the number of rows in the second matrix.

If A= a11 a12  B = b1then A. B = a11b1 + a12b2  a21 a22 b2a21b1 + a22b2

Matrix by matrix multiplication is not commutative, **A.B ≠ BA**

Example: Given the matrices A = 2 3 B = 4 1 find AB.

5 7 8 9

2 3 4 1

AB = x

5 7 8 9

AB = 2×4 + 3×8 2 x1 + 3×9 8+24 2 +27 =

5×4 + 7×8 5×1 + 7×9 20 +56 5+63 AB = 32 29

69

**EVALUATION**: 1. Find AC given the matrices A = -3 2 5 C = 4 7 2

0 1 8 3 -5 1

9 0 4

2. Given that A = 12 -8 B = 1 4 and C = 9 -5

3 6 2 8 15 10 Find I. A + B II. B – C III. 2A – B + 3C

**TYPES OF MATRICES**:

***Equal matrices***: Two matrices are said to be equal if corresponding elements are equal and the matrices are of the same order.

***Square matrix***: Is a matrix having the same number of rows and columns. E.g 2 x 2, 3 x 3,and so on

***Diagonal matrix:*** This is a square matrix with all elements zero except those on the main diagonal.

2 0 0

0 5 0

0 0 8

***Identity matrix:*** It is also called unit matrix and is a diagonal matrix in which the elements on the main diagonal are equal to one (1). It is denoted by **I.**

2 x 2 ,**I =1 0 3 x 3, I = 1 0 0**

**0 1 0 1 0**

### 0 0 1

***Null matrix***: Is a matrix whose elements are zero. It is denoted by **o.** i.e

0 0 0

0 0 0

0 0 0

***TRANSPOSE OF A MATRIX:*** The matrix obtained by interchanging the rows and columns of a matrix is called the transpose matrix. If **B** is the original matrix, its transpose is denoted by **BT.**

If A = 2 5 , then AT = 2 -8 7

-8 3 5 3 4

7 4

**EVALUATION**: If A = 1 2 3 and B = 7 10 find I A.B and (A.B)T

4 5 6 8 11

9 12

### WEEKEND ASSIGNMENT

Given that A = 4 2 3 B = 1 8 9

5 7 6 3 5 4

1. Find A + B. A. 5 10 12 B. 3 6 12

8 12 10 2 -2 2

1. Find A – B A. -3 -6 -6 B. 3 -6 -6

-2 2 -2 2 2 2

1. Find (A + 2B) T A. 6 18 21 B. 6 11 C. 6 18

11 17 14 18 17 21 11 21 14 17 14

1. Find A2 – 4I
2. Find BA.

### THEORY

1. Given that P = 1 5 find 2p2 – 3p + 5I

-4 2

1. 1 3 2 2 4 – 3 3 1 6

If P = 8 -4 4 Q = 3 8 4 R = 4 3 2

7 3 5 -1 3 6 2 – 1 1

Find (a) 5P + 2Q (b) 4Q – 2R (iii) 2P + Q + 3R (iv) PR

### Completing the square

To make a given expression a perfect square, the quantity to be added is the square of half of the coefficient of x ( or whatever letter is involved).

Examples:

In each of the following, add the term that makes the given expression into a perfect square , then write the result as the square of a bracketed expression.

1. g2 – 4 2/3 g
2. k2 – 11/3 k 3. m2 + 3mn.

Solutions. g2 – 4 2/3 g the coefficient of g is – 42/3 = –14/3 half of –14/3 = ½ x –14/3 =-7/3

Square of half of coefficient of g = (-7/3)2 = 49 ( + 54/9)

9

:. 49 must be added to the given expression to make it a perfect square

9

:. g2 -4 2/3 + 49/9 = ( n – 7/3) k2 – 1 1/3 k. the coefficient of k is ……..11/3 – –4/3 Half of –4/3 = ½ x –4/3 = –2/3

Square of half of coefficient of k = ( –2/3)2 = + 4/9.

:. 4/9 must be added to the given expression to make it a perfect square

:. K2 – 1 1/3 k + 4/9 = ( k – 2/3)2

1. m c xx + 3mn the coefficient of m is 3n .

half of + 3n =1/2 x + 3n = + 3n

2.

EVALUATION.

In each of the following add the term that makes the given expression into a perfect square . Write the result as the square of a bracketed expression

1b2 – 4/5b

2.u2 – 1 3/5 u

WEEKEND ASSIGNMENT

1.Find the value of k such that x2 + 5x +k is a perfect square.

(a) 2 ½ (b) 4 ¼ ( c) 6 ¼ ( d) 25 (e) 100.

1. What must be added to n2 + 1 1/s n to make it a perfect square?

(a) 3/5 (b) 1 1/5 ( c) 16/ 9 ( d)  9/16 ( e ) 1 ¼

1. Solve the equation

( x + 1 ¼ )2 = 1 9/16

( a) 2 ½, 0 ) (b) (0,2) ( c ) ( 0, – ) ( d ) ( 0, -2 ½ ) (e) ( 3, 1 ½ ).

4.Solve the equation

( x +1/3 )2 =4/9

( – 1/3, 1) (b) (1, 1/3 ) ( c) (2, 2/3 ) (d ) (-2/3, 3) (e )(1/3, -1).

5. What must be added to v2 – 3/4 v to make it a perfect square?

(a) 9/64 (b) 3/8 ( c) –3/8 (d) 7 1/9 (e) 2/9

Theory

In each of the following, add the term that makes the given expression into a perfect square. Then write the result as the square of a bracketed expression:

ii. u2 – 1 3/5u

2. a2 – 6ad

**TOPIC: SOLUTION OF QUADRATIC EQUATION & SYMMETRIC PROPERTIES OF THE ROOT OF QUATION EQUATION**

### CONTENT

Method of Factorization

Completing the square method.

Quadratic formula

Sum & Product of Roots of a Quadratic Equation

Symmetric Properties of Roots

### Method of Factorization

A quadratic equation is an expression of the form ax2 + bx + c = 0 in which a, b & c are numerals; and also the highest power of x is 2 & that the power of x will neither be fractions nor negatives. Quadratic equations can be solved using the method of factorization, completing the square, quadratic formula& graphical method

Steps in solving quadratic equation: (1)examine the middle term whose power of x is 1.

(2) Find the product of the first & last term. (3) Find two terms whose sum is equal to the middle term & product is equal to the value of the product of the first & last term (4) Replace the middle term by two the two terms in step 3. (5) Factorize the first two & last two terms (6) equate the linear factors to zero to find the value of x.

Example – Solve by factorization: X2 + 7X + 10 = 0

Solution

X2 + 7X + 10 = 0

X2 +2X + 5X + 10 = 0

X(X + 2) + 5(X + 2) = 0

(X+2) (X + 5) = 0

X + 2 = 0

X = -2 OR

X + 5 = 0

X = -5

Hence X = -2 or -5

### WEEK 5

Factorization of Quadratic Expressions.

Factorize the following:

1. 6a2 + 15a + 9
2. 6a2 – 19ax – 36x2
3. (5x – 1) ( x – 3) – ( x – 5) ( x – 3)
4. 35 – 2b – b2
5. x2 – y2 + ( x + y ) 2
6. 25a2 – 4 ( a – 2b ) 2.

Solutions.

6a2 + 15x + 9.

Since 3 is a common factor to all the terms, first take out 3 as the common factor:

6a2 + 15a + 9 = 3 (2a2+ 5a + 3)

= 3 (2a2 + 2a + 3a + 3)

= 3 (2a (a+1) + 3 (a+1)

= 3 ( (a+1) (2a+ 3)).

Hence.

6a2 + 15a + 9 = 3 (a+ 1) (2a + 3)

= 3 (a+1) ( 2a + 3)

2. 6a2 – 19ax – 36x2

1st step: Find the product of the first and last terms.

6a2 x -36x2 = -216a2x2

2nd step: Find two terms such that their products is – 216a2x and their sum is -19ax ( the middle term).

Factors of -216a2x2 sum of factors .

1. +27ax and -8ax + 19ax
2. -27ax and +8ax – 19ax
3. +9ac and -24ax – 15ax
4. -9x and 24ax + 15ax.

Of these only b gives the required result.

3rd step: Replace -19ax in the given expression by -27ax and 8ax then factorize by grouping:

6a2 – 19ax – 36x2

= 6a2 – 27ax + 8ax – 36x2

= 3a (2a – 9x) + 4x (2a-9x) =(2a – 9x ) ( 3a + 4x) hence,

6a2 – 19ax – 36x2 = (2a -9x) (3a + 4x)

(5x – 1 (x -3) – (x -5)(x – 3)

= (x – 3) ( 5x -1) – (x – 5)

= ( x -3) ( 5x – 1 – x + 5 )

= ( x – 3) ( 5x – x + 5 – 1)

= ( x – 3) ( 4x + 4)

= ( x – 3) + ( x + 1)

= ( x -3) 4(x+1)

= 4(x-3) (x+1) 4. 35 – 2b –b2  or – b2 – 2b + 35

= -b2 – 7b + 5b + 35

= -b (b+7) + 5 (b + 7)

= (b+7) ( -b + 5) = (b+7) ( 5-b) or (7+b) ( 5 – b)

5. x2 – y2 + (x + y)2

Since

X2 – y2 = (x)2 – (y)2 = ( x + y) ( x –y) then x2 – y2 + (x + y)2 = (x + y)(x-y) + (x+ y)2

= (x +y)( x-y + (x + y)

= (x + y ( x –y + x + y)

= ( x + y ) ( x + x

= (x + y) (2x)

(x +y ) (2x) = 2x ( x +y)

EVALUATION.

Factorize the following

1. m2 – 15mm – 54n2
2. 8a2 – 18b2
3. If 17x = 37 52 – 3562

Find the value of x

ASSIGNMENT

Factorize the following :

1. a2 + 5ab – 36b2
2. r2 – 25
3. 10p2 – 41p – 45
4. 12m2 – 4mn -5n2
5. 12a2b2 + 11ab – 5.

Theory.

Factorize the following

1. 8c2 – 14c – 9
2. x2y2 – xy – 30.

If I is a root of the equation 5x2 + kx – 3 = 0

Find the other root.

1. Given that ½ and -3 are the roots of the equation 0 = ax2 + bx + C, find a, b,c, where a, b and c are the least possible integers.

Solutions

1. Since 2 ½ is a root of the equation, then (x – 2 ½ ) is a factor, similarly, if -1 is a root of the equation then (x – (1-1) is a factor. i.e (x + 1) is a factor. Thus, the required equation is : (x – 2 ½ ) ( x + 1) = 0

x (x + 1) – 2 ½ ( x + 1) = 0 x2 + x – 2 ½ x – 2 ½ = 0 x2 – 1 ½ x – 2 ½ = 0.

X2 – 3/2 x – 5/2 =0

2x2 – 3x – 5 =0

2. Since I is a root of the given equation

5x2 + kx – 3 = 0

then it must satisfy the given equation.

Substitute I for x,

5(1)2 + k(1) – 3 = 0

5 x 1 + k – 3 = 0

5 + k – 3 = 0

\5-3 + k = 0 2 + k = 0 k = 0-2 k = -2.

Hence the given equation becomes

5x2 – 2x – 3 = 0 by factorization method, this quadratic equations can be solved as follows :

5x2 – 2x – 3 = 0

5x2 – 5x + 3x – 3 = 0

5x (x -1) + 3 ( x -1) = 0

(x-1) ( 5x + 3) = 0 (x -1) = 0 or 5x + 3 = 0

i.e. x = 0+ 1 or 5x = 0-3 x = 1 or 5x = -3

x = 1 or x = –3/5.

Hence the other roots of the given quadratic equation is x = 3/5.

2. Since the given roots of the equation 0 = ax2 + bx + C are ½ and -3, then the factors are (x – ½ ) and (x – c – 3)

i.e. ( x – ½ ) and ( x + 3)

Thus, the required equation is

(x – ½ ) ( x + 3) = 0

x(x+ 3) – ½ (x + 3) = 0 x2 + 3x – ½ x – 3/2 = 0

2x2 + 6x – x – 3 = 0

2x2 + 5x – 3 = 0

Comparing this with the given equation ax2 + bx + C = 0

Then

A= 2, b= 5, c = -3.

EVALUATION

I. 7 and -3 are the roots of the quadratic equation x2 +kx -21 = 0,what is the value of k? 2. -5 is a root of the

ASSIGNMENT

1.Find the quadratic equation whose roots are 2 and -3.

(a) (x2 + x -6 (b) x2 – x -6 (c ) x2 + 5x – 6 (d) x2 – 5x + 6 (e) x2 + 3x – 6.

1. (2x + 3) is a factor of 6x2 + – 12. The other factor is ………….

(a) (x + 6) (b) (2x – 3) (c) (3x + 4) (d) (3x – 4) (e) (4x – 9).

1. Find the roots of the equation x2 + 12x – 28 = 0 the

Examples

* 1. The electrical resistance R ohms of a wire varies directly as the length cm and inversely as the square roots of the diameter d cm3.
  2. Express d in terms of l,R and the constant of variation k.
  3. Find the value of d, correct to 2 decimal places, when 1 = 15cm, R = 0.13ohms and k = 1.25 x 10-3.
  4. V varies jointly as the square of x and inversely as y, if V =18 when x = 3 and y =4, find V when x = 6 and y =9.

Solutions.

The greater of the two roots is …………………………………

(a) -14 (b) -2 (c ) 2 (d) 14

1. What is the product of the roots of the equation x2 -3x + 2 =0?

(a) -3 (b) -1 (c) 2 (d ) 3 (e)5

1. If x2 – 10x -24 =0, the sum of the roots of the equation is ………….

(a) – 10 (b) 10 (c ) -24 (d) -2

THEORY

* 1. find the quadratic equation whose roots are (3 + √5).
  2. If 1 is root of the equation 5×2 + kx – 3 =0, find the other root.

Quadratic equation n2 – 8n – 65 =0, what is the other root of the equation?

**Topic: Solution of Quadratic Equation by Factorization Method** Examples:

Solve the following quadratic equations by the factorization methods.

m2 = 11m + 42 2x2 – x – 21 =0 2p2 + 11p = 30.

Solutions. (1 ) m2 = 11m + 42

then m2 – 11m – 42 =0

1st step: Find the product of the first and last terms :

m2 x – 42 =-42m2

2nd step:find two terms such that their product is -42m2 and their sum is -11m ( the middle term).

Factors of -42m2 sum of factor s

1. + 3m and – 14m – 11m
2. – 6m and +7m + 11m

( c) -6m and + 7m +m

(d) +6m and -7m -m

Out of these, only (a) gives the required result.

3rd step: Replace -11 in the given expression by +3m and -14m. then factorise by grouping: m2 -11m -42 = 0.

M2 + 3m – 14m – 42 =0 M(m+ 3) (m – 14) = 0 M + 3 = 0 or m – 14 =0

i.e m = 0 -3, or m = 0 + 14.

M = -3 or m = 14.

2. 2x2 –x -21 =0

1st Step: Product of first and last terms = 2x2 x -21

= – 42x2

2nd step: Find two terms such that their product is 42x2 and their sum is –x (the middle term).

Factors of-42x2 sum of factors

1. + 2x and -21x -19x
2. -2x and 21x +19x

( c)-3x and + 14x + 11x

(d) + 3x and -14x -11x

( e) +6x and -7x -x

(f) -6x and + 7x +x out of these, only (e)gives the required result.

3rd step: Replace –x in the given expression by +6x and -7x. then factorize by grouping

2x2  – x -21 =0

2x2 +6x -7x – 21 = 0 x ( x + 3) -7 ( x + 3 ) = 0 ( x + 3) ( x-7) =0 either x + 3 = 0 or 2x -7 =0

i.e x = 0-3 or 2x = 7 x = -3 or x =7/2 x = -3 or x = 3 ½ .

3. 2p2 + 11p = 30.

2p2 + 11p – 30 = 0

1st step: Product of the first and last terms

2p2 x -30 =- 60p2

2nd step: find two terms such that their products is -60p2 and their sum is +11p (the middle term )

Factors of +11 sum of factors

1. + 6p and – 10p -4p
2. -15p and +4p +4p( c) -15p and +4p -11p

(d) +15p and -4p +11p.

Out of these, only (d) gives the more required solution. Replace + 11p by +15p and -4p in the given expression . 2p2 + 15p -4p -30 = 0 p ( 2p + 15) – 2 (2p + 15) = 0 (2p + 15) (p – 2) = 0 either 2p+ 15 = 0 or p -2 = 0

2p = 0 – 15 or p = 0 + 2 2p = -15 or p = 2 p = -15 or p = 2 p = –15/2 or p = 2 p = -7 ½ or p = 2.

EVALUATION.

Solve the following quadratic equations by factorization method.

4e2  – 20 e + 25 = 0

4a2 – 11a = 3

ASSIGNMENT.

1. If x2 – 10 x – 24 = 0, then x = 12 or ……………
   1. -3 (b) -2 (c) -1 (d) 1 (e) 2
2. if x2 + Kx + 16/9 is a perfect square, then K = ………………..
   1. 1/3 (b) 2/3 (c) 4/2 (d) 8/3 (e) 16/3
3. What is the sum of the roots of the equation x2 -3x +2 =0?
   1. – 3 (b) -1 ( c) 2 (d )3 (e) 5
4. Find the roots of the equation x2 + 12x – 28 = 0

The greater of the two roots is ………….

(a) -14 (b) -2 (c ) 2(d)7 (e ) 14.

5. What are the factors of 6×2 + x – 12?

A.(2x+ 3)( x + 6) B. (2x +3) ( 2x – 3) C. (2x +3)(3x + 4) D. (2x + 3) (3x -4)

E. (2x + 3 ) ( 4x -9).

THEORY.

Solve the following equations”

1. 4y2 – 28y + 49 =0
2. 2x2 + 11x + 5 = 0.

**Completing the square method Given a quadratic equation, a**  **+bx** **=0 (where a, b and c are constants) a**  **+bx =-c (subtract c from both sides)**

**+**  **(divide both sides by a)**

**The coefficient of x=**

**Divide**  **by 2 =**

**Square**  **and add to both sides i.e**

**+**  **=**  **i.e.**

**(X +**  **=**  **or**

**X +**

**X +**

### X =

**X =** **. Q.E.D.**

4Y + 4 = 0

SOLUTION

4Y = 4 (Re-arrange the equation)

(Divide through by coefficient of )

4Y+ ( = . Half Y, square it and add it to both sides.

=

(Y = 0 Y 2 =

Y = 0 +2 Y = +2 or 2

OR using method of difference of two squares

(Y 2)(Y+2) = 0

Y 2 = 0 or Y + 2 = 0

Y = 2 or Y = 2.

Example – Solve the equation using completing the square method**.**

1. X2 – 8x + 3 = 0
2. 3x2 – 5x – 7 = 0

Solution

1. x2 – 8x + 3 = 0

Step 1**:** Take C to the other side x2 – 8x = -3 Step 2**:** Divide through by the coefficient of x2

x2 – 8x = -3

1 1 1

Step 3**:** Divide the coefficient of x by 2, then square and add it to both sides.

Co efficient of x = 8

: 8 = 4

2

42 = 16

=x2 – 8x + (-8)2 = -3 + (- 8)2

2 2

x2 – 8x + (-3)2 = -3 + (-4)2

(x – 4)2 = -3 + 16 (x – 4)2 = 13 x – 4 = + 13

x = +4 + 13 x = 4 + 3.61

x = 4 – 3.61 or 4 + 3.61

0.39 or 7.61

2. 3x2 – 5x – 7 = 0 3x2 – 5x = +7

3x2 – 5x = 7

3 3 3 x2 – 5x = 7

3 3 x2 – 5x + (-5)2 = 7 + (-5)2

3 6 3 6

(x – 5)2 = 7 + 25

6 3 36

(x – 5)2 = 84 + 25

36

(x -5)2 = +109

6 36

x = 5 + +109

6 6

x – 5 = +10.44

6 6

x – 5 = +10.44

6 6 x = 5 + 10.44 or 5 – 10.44

6 6 x = 15.44 or -5.44

6 6 x = 2.5.573 or – 0.906 = 2.57 or – 0.91

m = 7 + 2.24

2

m = 7 + 2.24 or 7 – 2.24

2 2 m = 9.24 or 5.24

2 2

m = 4.62 or 2.62 2. 3x2 + 7x -3 = 0 3x2 + 7x = 3

3x2 + 7x = 3

3 3 3

x2 + 7x = 1

3 x2 + 7x + (7)2 = 1 + (7)2

3 6 6

(x + 7)2 = 1 + 49

6 36

(x + 7)2 = 1 + 36 + 49

6 36

(x + 7)2 = 85 6 36 x + 7 = +85

6 36

x + 7 = +85

6 6 x + 7 = +9.22

6 6 x = -7 + 9.22 or -7 – 9.22

6 6

= 0.37 or 2.70

– 16 .22 6 6

or

X= 2.22 6

Examples

Solve the following equations by completing the square method.

1. n2 – 12n + 1 = 0 2. y2 +7y – 30 =0

3. m2 -7m + 11 = 0

Solutions

1. n2 – 12n + 1 = 0 n2 – 12n = 0 – 1 n2 – 12n = -1.

Add to both sides the square of –12/2 ( -6).

n2 – 12n ( -6) 2 = -1 + (-6)2

(n – 6) 2  = -1 + (-6) 2

(n – 6) 2 =+ 35.

Take square root of both sides. n-6 = ± √35.

i.e n = ± √35 + 6 n = + √35 + 6 or n = -√35 + 6 n = 6 + √35 or n = 6 – √35 n = 6 + 5.916 or n = 6 – 5.916 n = 11.916 or n = 0.084.

1. e . n = 11.92 or n = 0.08 to 2 decimal places.2.y2 +7y – 30 = 0 y2 + 7y = 0 + 30 y2 + 7y = 30

Add to both side the square of 7/2 y2 +7y + ( 7/2)2 = 30 + (7/2) 2

i.e. ( y +7/2) 2  = 30+ 49

1 4.

( y + 7/2) 2 = 120 + 49

4

( y + 7/2)2 = 169

4.

Take square root of both sides : :. Y + 7/2 = ± √169/4/ y + 7/2 = ± 13/2

i.e y = ± 13/2 –7/2 .e y = +13/2 – 7/2 or y = – 13- 7

2.

y = +6 or y = -20

2 2

y = +3 or y = -10.

3.m2 – 7m + 11 = 0 m2 – 7m = 0 – 11 m2 -7m = -11

Add to both sides the square of –7/2

M2 – 7m ( –7/2)2 = -11 + (-7/2)2

( m – 7/2)2 = -11 + 49

4

( m – 7/2)2 = -44 + 49

4.

( m – 7/2)2 = 5/4 Take square root of both sides:

m –7/2 = ± √5/4 m= ±√5/4 + 7/2 m=± √+ 7/2

i.e. m = + √5 + 7 or m = – √5 + 7

2 2 2 2.

m = + 2.236 +7 or m = – 2.236 + 7

2 2 2 2

m=+1.118 + 3.5 or m = -1.118 + 3. 5 m = + 4.618 or m = 2.418

i.e m = 4.62orm = 2.42. to 2 decimal places EVALUATION.

Solve the following method of completing the squares:

* 1. x2 – 8x – 1 = 0
  2. p2 -10p + 15 = 0

WEEKEND ASSIGNMENT

1. The greater of the two roots of the equations (2x -5) (3x +10 ) = 0 is (a) – 50 (b) -10 ( c) -3 (d) 2 ½ ( e) 5.
2. If one of the roots of the equation ( n-13)2 = 9 is 10, what is the other root?

(a) 4 (b) 16 ( c) 22 (d) 68 ( e ) 94.

1. Find the value of k such that x2 + 5x +k is a perfect square.

(a) 2 ½ (b) 4 ¼ ( c ) 6 ¼ ( d) 25 ( e) 100.

1. If x2 – 10x = 24, what is the values of x ?

(a) 12,-3) (b) ( -2, 12) ( c) -1, 12) (d) ( 1, 12) (e) (2,12).

1. What must be added to a2 + 3/5a to make it a perfect square?

(a) 3/10 (b) 6/5 ( c) 9/100 (d) 100/9 ( e) 36/5

THEORY

Solve the following equations, giving answers correct to 1 decimal place.

x2 + 14x + 8 = 0 2p2 +9p – 30 =0 **Quadratic Formula**

Quadratic formula is derived by using the method of completing the square. Considering the general form of quadratic equation: ***ax2 + bx + c = 0* X =**  **The required quadratic formula.** Example – Solve x2 + 3x – 2 = 0 using formula method

+

a = 1 6 = 3 C = – 2 x = -b b2 – 4ac

+

2a x = -3 32 – 4(i) (-2)

+

2(i)

x = -3 9 + 8

2

x = -3 17

+

2

x = – 3 4.12

2 x = -3 + 4.12 or -3 – 4.12

2 2 x = 0.56 or – 3.56

2. m2 – 7m + 11 = 0

Solve using formula method.

m2 – 7m + 11 = 0

+

a = 1 b = – 7 c = 11 m = – b b2 – 4ac

+

2a m = – (-7) 72 – 4(i) (ii)

+

2(i) m = 7 49 – 44

+

2 m = 7 5

+

2 m = 7 2.23 2 m = 7 + 2.23 or 7 – 2.23

2 2 m = 4.62 or 2.62

### EVALUATION

1. 3x2 + 7x – 3 = 0 solve using formula method
2. Using completing the square and formula method solve 3x2 – 12x + 10 = 0

### Sum & Product of Roots of a Quadratic Equation

The expression for sum & product of roots of quadratic equation is gotten from the general expression of quadratic equation. If the distinct roots are α and β, then α + β = -b/a (sum of roots) αβ =c/a (product of roots)

Example 1 – find the sum and products of 2x2 + 3x – 1 = 0

Solution 2x2 + 3x – 1 = 0 a =2, b = 3, c = -1

Let α and β be the roots of the equation, then α+β= -b/a= -3/2 αβ = c/a = -1/2

Example 2 – find the sum and products of 3x2 – 5x -2 = 0

Solution 3x2 -5x -2 =0 a= 3, b= -5, c= -2 let α and β be the roots of the equation, then α+β= -b/a = 5/3 αβ= c/a= -2/3

**NB:** The quadratic equation whose root are α and β is

**(X – α )(X – β) = 0**

### X2 – (α +β)X + αβ = 0

Example – Find the quadratic equation whose roots are 3 & -2 Solution α=3 and β=-2 α+β = 3 + (-2) = 1 αβ = 3 x -2 = -6

X2 – (α +β)x + αβ = 0

X2 – (1)X + (-6) = 0

X2 – X -6 = 0

### Symmetric Properties of Roots

Certain relations involving α and β can also be determined from α+β andαβ even when we do not know α and β distinctly. Such relations are usually said to be symmetric. They are symmetric in the sense that if α and α are interchanged, either the relation remains the same or is multiplied by -1

Example – If α ≠ β, determine whether or not each of the following is symmetric

(a) α+β (b) αβ (c) α2 + β2 (d) α2 – β2 (e) 3α +2β (f) α3 + β3

Solution

α+β = β +α , therefore α+β is symmetric αβ =βα, therefore αβ is symmetric

α2 + β2 = β2 + α2, therefore α2 + β2 is symmetric α2 – β2 = – (β2 – α2), thereforeα2 – β2 is symmetric

3α +2β ≠ 3β + 2α since α ≠ β, therefore 3α +2β is not symmetric α3 + β3 = β3 + α3,  therefore α3 + β3 is symmetric

### EVALUATION

1. Find the quadratic equation whose roots are ½ and 5
2. Find the sum & product of roots of the equation 3x2 – 5x – 2 = 0

### ASSIGNMENT

1. Solve for x ; x2 – 9 = 0 (a) 3, 3 (b) 3, -3 (c) 9, 3 (d) -3, -3
2. Find the sum and products of 2x2 + 3x – 1 = 0 (a) -3/2, ½ (b) 3/2 , -1/2 (c) -3/2, -1/2 (d) 3, 2
3. Solve for x : 6x2 – 13x + 5 = 0 (a) 5/3, ½ (b) -5/3,1/3 (c) 5/3, -1/3 (d) 3/5, 1/3
4. Factorize 6x2 + 5x -6 (a) (3x – 2) (2x + 3) (b) (3x + 2) (2x + 3) (c)(3x – 2) (2x – 3) (d) (2x -3) (2x+ 2)
5. Find the sum and products of x2 – 4x – 3 = 0

### Theory

1. Solve S = ut + ½ at2 using the completing the square method
2. Find the quadratic equation whose roots are ¾ and ½

### WEEK 6

**MENSURATION – Surface Area and Volume of Spheres and Hemispherical Shapes MENSURATION** Is defined as a branch of Mathematics that deals with measurement, especially the derivation and use of algebraic formulae to measure the areas, volumes and different parameters of geometric. Examples are cylinder, cone, cuboid, rectangular prism, rectangular based pyramid, total surface area of cylinder, cone and their volume.

FORMULAE

SHAPES AREA SURFACE TOTAL SURFACE VOLUME

AREA AREA (T.S.A)

(C.S.A)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| CONE | rl | rl | l + r) |  |
| CYLINDER |  | 2 rh | 2 rh+2 |  |
| CUBOID |  | 2(lb+lh+bh) |  | l.b.h |
| TRIANGULAR PRISM |  |  | Sum of areas of all surfaces | Area of cross section x height |
| RECTANGULAR PYRAMID |  |  | Sum of all four triangular faces + base area | x base area x height |
| CUBE |  |  | 6 |  |
| SPHERE |  | 4πR2 |  | πR3 |

HEMISPHERE 2πR2 3πR2 4πR3

**SPHERE A HEMISPHERE**

### Volume of a Sphere

A **sphere** is a solid in which all the points on the round surface are equidistant from a**fixed point,** known as the center of the sphere. The distance from the center to the surface is the **radius**.

**Volume of sphere** = where *r* is the radius.

How to find the volume of a sphere? What is the volume of air in the ball?

**Volume of a hemisphere**

A **hemisphere** is half a sphere, with one flat circular face and one bowl-shaped face.

**Volume of hemisphere**  where *r* is the radius

#### Spheres

**What is a sphere?**

A **sphere** is a solid with all its points the same distance from the center. The distance is known as the radius of the sphere. The maximum straight distance through the center of a sphere is known as the diameter of the sphere. The diameter is twice the radius.

**How to find the volume of a sphere?**

The volume of a sphere is equal to four-thirds of the product of pi and the cube of the radius.

The volume and surface area of a sphere are given by the formulas:

where *r* is the radius of the sphere.

***Example:***

Calculate the volume of sphere with radius 4 cm.

***Solution:***

Volume of sphere

We can also change the subject of the formula to obtain the radius given the volume.

***Example:***

The volume of a spherical ball is 5,000 cm3. What is the radius of the ball?

***Solution:***

Example: Find the volume of a sphere with a diameter of 14 cm.